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Determination of Hydraulic Conductivity Using Analytical and Numerical Methods Applied to Well and Aquifer Tests†

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ABSTRACT

This study aims to determine the hydraulic conductivity of aquifers by analytical and numerical methods applied to water level data obtained from well and aquifer tests. Two wells 33 meters apart were drilled in an unconfined aquifer in Bursa Küçük Sanayi region by Neojen Mühendislik. In one of the wells, a total of 18 slug tests were performed using different volumes before and after well development. Bouwer-Rice and Dagan methods were used as analytical methods and MODFLOW software, which solves groundwater flow problems using method of finite differences, was used for numerical modeling. In the other well, pumping tests were performed. Hantush-Bierschenk and Rorabaugh methods were applied to the data from step-drawdown tests and discharge-drawdown curves were obtained. In addition to these, recovery test and multirate test were performed to determine the hydraulic conductivity. In MODFLOW, simulations were made for various cases related to grid resolution and specific yield. It was observed that well development significantly contributes to well performance and that the volume used in slug tests does not affect the results. In addition, different pumping tests modeled in MODFLOW yielded similar values of hydraulic conductivity.

Keywords: *Hydraulic conductivity, slug test, recovery test, multirate test, step-drawdown test, MODFLOW*

1. INTRODUCTION

Water, as the essential and indispensable need of human beings, plays a vital role in domestic consumption, agriculture, energy production, industry as well as in social and economic development. In addition, due to the limited amounts of water resources and their strategic value, a fair and efficient distribution of water is crucial. Today, climate change leads to shortage of water while the demand from limited resources increases with population. As an important resource, groundwater should be paid particular interest. As a part of groundwater resources assessment methods, researches on the identification of hydraulic characteristics of aquifers gain importance. Determination of an aquifer's

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parameters means conceiving the behavior and properties of that particular aquifer which will in turn lead to an easier utilization of groundwater.

This research aims to determine the hydraulic conductivity, an aquifer parameter affecting groundwater flow dynamics, by using well tests in unconfined aquifers. For this purpose, two wells were drilled in an unconfined aquifer in Bursa Küçük Sanayi by Neojen Mühendislik. In one of the wells, slug tests were performed using 3 different volumes; 7 ,10 and 19 L. With each volume, 3 tests were performed before well development and 3 tests after well development, which makes a total of 18 tests. Bouwer-Rice and Dagan methods [1] were used as analytical methods and MODFLOW [2] was used for numerical modeling. Both were applied to each test in order to determine the hydraulic conductivity. The effect of well development, a process for cleaning the well, on hydraulic conductivity of the aquifer surrounding the well was analyzed together with the well performance. In the other well, 5 pumping tests, one recovery test and one step-drawdown test were performed. All of the pumping tests were modeled in MODFLOW, and simulations for various conditions based on grid resolution and specific yield were made. The condition that best represents the aquifer was tried to be determined. Using recovery test and multirate test, hydaulic conductivity was calculated. Hantush-Bierschenk and Rorabaugh methods were applied to the step-drawdown test results and discharge-drawdown curves were obtained. As the two wells drilled in the field were far enough not to affect each other, Neuman curve-fitting method [3] and similar methods based on observation wells were not applied.

2. FIELD STUDY

Two wells were drilled in an unconfined aquifer in Bursa Küçük Sanayi region (Fig. 1). The first well was 27 meters deep and pumping tests were carried out in this well. The second well was 20 meters deep and it was used for slug tests. The distance between the two wells was 33 meters.

Both wells were drilled using mud rotary drilling method [4]. In this study, both wells were drilled using a drill diameter of 250 mm. In the first well, a total 7 PVC pipes with a diameter of 140 mm were placed. Each pipe was 4 m long where the bottom one was unfiltered and had a cap and the others were filtered. The impermeable solid layer was reached at a depth of 27 m. In the second well, 5 PVC pipes with a diameter of 140 mm and each with a length of 4 m were placed. The bottom one was unfiltered and had a cap and the others were filtered. The impermeable solid layer was reached at a depth of 20 m. After installing the pipes, graveling was carried out. The space between PVC well pipe and the ground was filled with gravel. The purpose of this is to prevent plugging of filters during withdrawal from or injection into the well. During graveling, plenty of water is injected into the well in order to wash out the mud. Next stage is well development which is cleaning of the well. For this purpose, a column pipe with a diameter of 63.5 mm and an air pipe with a diameter of 32 mm were placed one inside another into the well pipe (Fig. 2). During well development, air was pumped with a compressor into the air pipe. Column pipe was connected to a valve on the surface in order to evacuate the water rising in the column pipe during well development. The process was repeated in a cyclic manner until fresh water was observed from the well with valve fully open and then fully closed. When the valve is

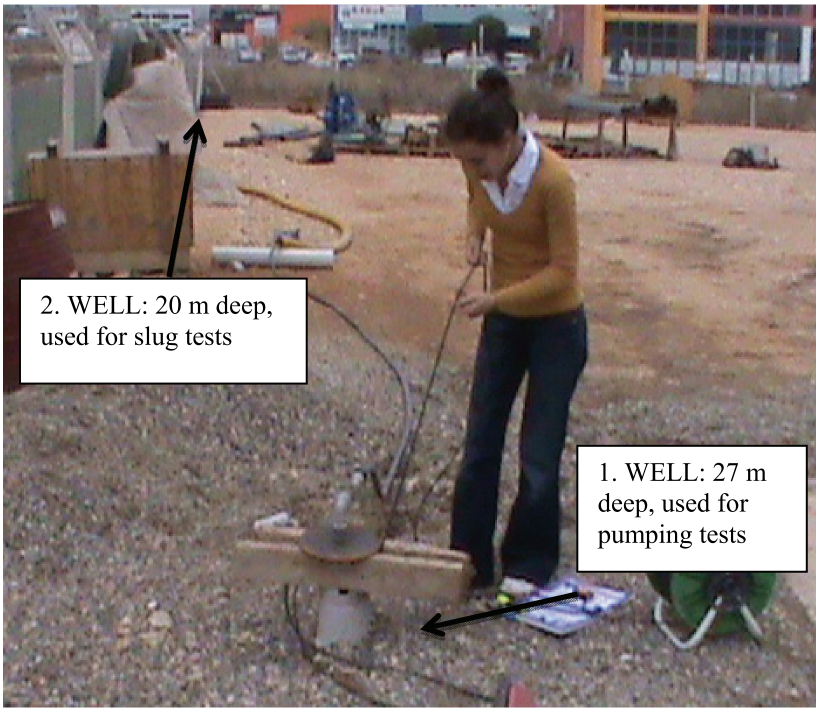

Figure 1. Wells in the field

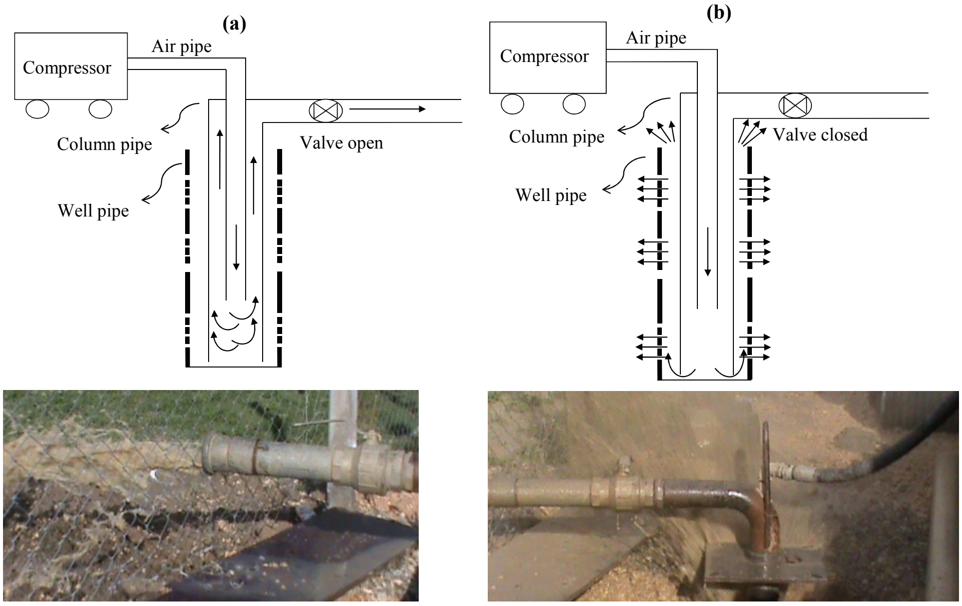

Figure 2.(a) Valve is open (b) Valve is closed

open, pressurized water rises between the air pipe and column pipe and flows out from the surface extension of the column pipe (Fig. 2a). When the valve is closed (Fig. 2b), pressurized water can not rise in the column pipe, therefore, it will rise in the well pipe and leak into gravel pack through the filters. During the leakage, the material in the gravel pack is aligned largest to smallest from the pipe outwards. As the well pipe is not connected to an outlet pipe, pressurized water rising in the well erupts to the surface (Fig. 2b). The operation is repeated for an hour with the valve fully closed and for half an hour with the valve open in a cyclic manner until fresh water comes from the well. As a result, well development process was completed. The column pipe and air pipe were removed. The well became ready to use.

3. ANALYTICAL METHODS USED IN AQUIFER TESTS

3.1. Slug tests

Slug tests are relatively faster to apply and economical. One single well is needed to perform the test and there is no need for pumping. The test does not take long time, however, the hydraulic conductivity obtained through this method represents a small area around the well. Basically, slug test is performed either by dropping a slug with a definite volume into the well or by sudden removal of a previously inserted slug from the well and the change in water level is observed. A definite amount of water can also be used instead of a slug. In this study, the test is carried out by pouring water instantaneously into the well (Fig. 3). Water level is recorded right from the beginning. Water level begins to decrease immediately and it is measured with time until water level becomes stable. This time period can vary from several minutes to days according to the type of soil. Two analytical methods used in slug tests to determine hydraulic conductivity are explained below.

3.1.1. Bouwer and Rice method

Bouwer & Rice Method [5], [6], [7] was originally designed for isotropic conditions. Ztolnik [7] enhanced the method for anisotropic conditions. Analytical solutions were developed for slug tests under certain assumptions, which are defined as follows [3].

1. Aquifer has an infinite extent on horizontal plane.
2. Aquifer is homogenous, isotropic and has constant thickness.
3. Density and viscosity of groundwater are constant.
4. Before the test, the water table or piezometric surface in the aquifer is horizontal.
5. Hydraulic head in the well suddenly changes at t=0 and change in the water table around the well is neglected.
6. Oscillations of water level in the well and linear and non-linear losses in the well can be neglected.
7. The well penetrates aquifer's saturated thickness partially or completely.
8. Well diameter is finite; therefore, storage in the well can not be neglected.
9. Water inflow to or outflowing from the well is transmitted horizontally.

The following formulas are defined for the analytical solution of the method (Fig. 3).

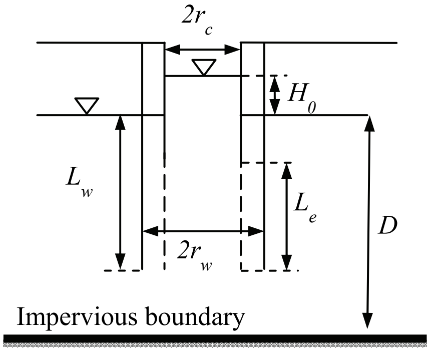

Figure 3. Condition following a sudden pouring of a definite volume of water into the well in an unconfined aquifer.

$$H_0 = \frac{V}{\pi\, \mathrm{r}_c^2} \tag{1}$$

H_0= Change in water level right after pouring water (at t=0) [L]

V= Volume of water poured [L^3]

r_c= Radius of the well casing [L]

$$K = \frac{r_c^2 \ln\left(\frac{R_e}{r_w}\right)}{2L_e} \frac{1}{t} \ln\left(\frac{H_0}{H(t)}\right) \tag{2}$$

K= Hydraulic conductivity [LT^{-1}]

r_w= Radius of gravel envelope [L]

R_e= Effective radial distance over which head is dissipated [L]

L_e= Length of the screen [L]

t= Time [T]

$H(t)$= Drawdown as a function of time [L]

R_e is also the characteristic distance from the well over which the average value of K is measured. However, R_e can not be known for a specific well. $\ln(R_e/r_w)$ is calculated as follows [5], [6].

a) If the distance between water table and base of well (L_w) is smaller than the saturated thickness of aquifer (D) (well partially penetrating the aquifer);

$$\ln\left(\frac{R_e}{r_w}\right) = \left[\frac{1.1}{\ln\left(\frac{L_w}{r_w}\right)} + \frac{X + Y\ \ln\left(\frac{D - L_w}{r_w}\right)}{\frac{L_e}{r_w}}\right]^{-1} \tag{3}$$

b) $L_w = D$;

$$\ln\left(\frac{R_e}{r_w}\right) = \left[\frac{1.1}{\ln\left(\frac{L_w}{r_w}\right)} + \frac{Z}{\frac{L_e}{r_w}}\right]^{-1} \tag{4}$$

Dimensionless X, Y, Z parameters depend on L_e/r_w values and obtained from the curves in Fig. 4. The greater the values of r_w and L_e, the greater the area of aquifer over which the hydraulic conductivity is determined [1]. Computation steps are as follows;

1. Mark $H(t)/H_0$ values on a logarithmic vertical axis and time (t) values on arithmetic horizontal axis. Draw a straight line through marked points.
2. X, Y, Z values corresponding to L_e/r_w values are read from Fig. 4.
3. Choose and apply the appropriate formula from case (a) or (b).
4. $H(t)$=0.368H_0 is chosen and the value of $\ln\left(H_0/H(t)\right)$ in Eq. (2) becomes 1. In order to read the necessary t value in the same formula, a correction is made to overcome the early time effect. The straight line is extended backwards till it intersects the y-axis ($[H(t)/H_0]$ axis). The point of intersection gives the $(H_0)_{new}/H_0$ ratio. This ratio is multiplied by $H(t)/H_0$=0.368 and a revised $[H(t)/H_0]_{new}$ value is obtained and the corresponding t_{new} value is read from the x-axis.
5. Finally, hydraulic conductivity is calculated from Eq. (5).

$$K = \frac{r_c^2 \ln\left(\frac{R_e}{r_w}\right)}{2L_e} \frac{1}{t_{new}} \tag{5}$$

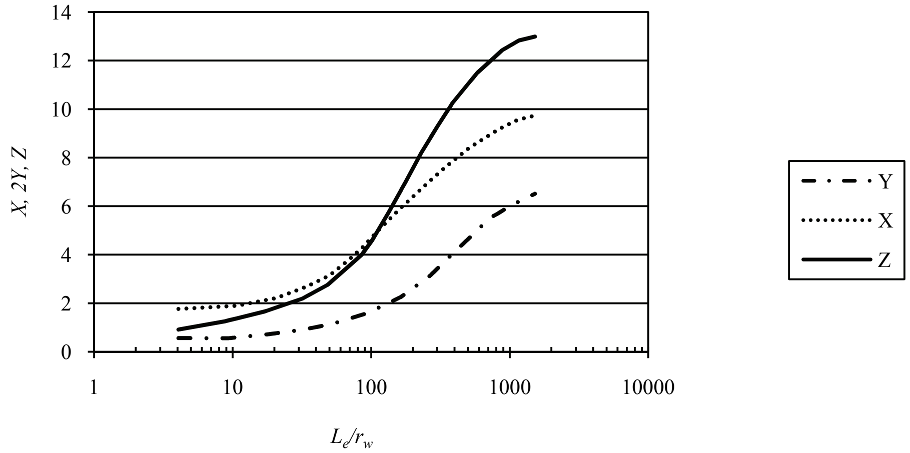

Figure 4. Dimensionless parameters X, Y, Z plotted as a function of L_e/r_w [4]

3.1.2. Dagan method

Dagan [8], takes Bouwer & Rice method as a base, however, it is slightly different: In Dagan method, there is no boundary condition, whereas in Bouwer-Rise method it is assumed that there is a constant head boundary condition at a specified distance from the well. The calculation steps are provided below;

1. Mark $H(t)/H_0$ values on a logarithmic vertical axis and time (t) values on arithmetic horizontal axis. Draw a straight line through marked points.
2. The straight line is extended backwards till it intersects the y-axis ($[H(t)/H_0]$ axis). The value on the y-axis is multiplied by $H(t)/H_0$=0.368 and a revised $[H(t)/H_0]_{new}$ value is obtained. The corresponding t_{new} value is read.
3. Using the value of anisotropy ratio K_z/K_r, the parameter ψ is calculated from the following equation (in general, K_z/K_r=1).

$$\psi = \frac{\sqrt{K_z/K_r}}{L_e/r_w} \tag{6}$$

4. Using the ψ value, the dimensionless flow parameter, P, for unconfined aquifers is obtained from either Table 1 or 2. Another table for confined aquifers is presented in [9].

Table 1. Tabulated values of the dimensionless flow parameter, P, used in Dagan method for wells screened below the water table (values for $L_e/D \leq 0.05$)[9]

	$(L_w+L_e)/L_e$				
ψ	8	4	2	1.5	1.05
0.2	0.646	0.663	0.705	0.756	1.045
0.1	0.477	0.487	0.505	0.531	0.687
0.067	0.409	0.416	0.429	0.446	0.562
0.050	0.367	0.373	0.385	0.397	0.491
0.033	0.322	0.325	0.335	0.352	0.414
0.025	0.294	0.297	0.305	0.322	0.370
0.020	0.276	0.278	0.287	0.301	0.342
0.013	0.247	0.249	0.255	0.269	0.300
0.010	0.230	0.231	0.238	0.250	0.276
0.0067	0.211	0.210	0.213	0.227	0.248
0.0050	0.198	0.199	0.201	0.213	0.230

Table 2. Tabulated values of the dimensionless flow parameter, P, used in Dagan method for wells screened below the water table (values for (L_w+L_e)=D) [9]

	L_e/D					
ψ	1	0.83	0.67	0.50	0.20	0.10
0.20	1.289	0.723	0.631	0.576	0.510	0.492
0.10	0.800	0.510	0.460	0.428	0.390	0.380
0.050	0.536	0.384	0.354	0.335	0.312	0.306
0.025	0.388	0.305	0.286	0.273	0.258	0.254
0.010	0.279	0.238	0.227	0.219	0.209	0.206

5. Finally, hydraulic conductivity is calculated from Eq. (7).

$$K_r = \frac{r_c^2(1/P)}{2L_e t_{new}} \tag{7}$$

3.2. Pumping tests

3.2.1. Step-drawdown test

Step-drawdown test is performed in one single well. The objective of the test is to obtain a discharge-drawdown curve that estimates drawdown corresponding to a given discharge which may be larger than the one used during the test. The test starts with a low pumping rate and when the drawdown approaches a constant value, the pumping rate is increased. The process continues for at least 3 steps. Each step lasts for about the same duration. Generally one step takes between 30 to 120 minutes depending on aquifer properties. This test was primarily studied by Jacob [10] and the following equation is proposed:

$$s=B(R_e,t)Q+CQ^2 \tag{8}$$

$B(R_e,t)=B_1(r_c,t)+B_2$ [TL^{-2}]

$B_1(r_c,t)$= Linear aquifer loss coefficient [TL^{-2}]

B_2= Linear well loss coefficient [TL^{-2}]

C= Non-linear well loss coefficient [T^2L^{-5}]

R_e= Effective radius of the well [L]

r_c= Actual radius of the well [L]

t= Pumping time [T]

s= Drawdown [L]

Rorabaugh [11], proposed the following form of Jacob equation. Eq. (9) is called Rorabaugh equation. P value is between 1.5 and 3.5 [12]. On the other hand, the value of $P=2$ as suggested by Jacob is also widely used.

$$s=BQ+CQ^P \tag{9}$$

There are two methods for finding the unknown parameters in Eqs. (8) and (9), namely, Hantush-Bierschenk method [13], [14] and Rorabaugh method [11].

3.2.1.1. Hantush-Bierschenk method

Hantush [13] and Bierschenk [14] proposed a method with the following assumptions [3]:

1. The aquifer is confined, leaky or unconfined.
2. The aquifer has an infinite areal extent.
3. The aquifer is homogeneous, isotropic and of uniform thickness.
4. Before pumping, the piezometric surface is horizontal over the area that will be influenced by the test.
5. The pumping rate at each step is held constant and greater than that of the previous step.
6. The well penetrates the entire thickness of the aquifer and receives water in horizontal direction.
7. The flow to the well is in an unsteady state.
8. The non-linear well losses can not be neglected and vary according to the expression CQ^2.

Hantush described the drawdown in the well at the nth step of step-drawdown test by applying the superposition principle to the Jacob equation (Eq. (8)):

$$s_{w(n)} = \sum_{i=1}^{n} \Delta Q_i B\left(R_{e,}\, t-t_i \right) + CQ_n^2 \tag{10}$$

$s_{w(n)}$= Total drawdown in the well during the nth step at time t [L]

R_e= Effective radius of the well [L]

t_i = Time at which the ith step begins [T]

Q_n = Constant discharge during the nth step [L^3T^{-1}]

Q_i = Constant discharge during the ith step [L^3T^{-1}]

$\Delta Q_i=Q_i-Q_{i-1}$

On a semi-logarithmic graph, time is placed to the horizontal logarithmic axis and the observed drawdown to the vertical axis. A straight line is fitted to the last portion of each step and then it is extended to the end of the following step. The drawdown increment values, $\Delta s_{w(i)}$, are read at a fixed time interval (Δt) from the beginning of each step as the difference between the observed drawdown value and the extension of the line from the previous step. Total drawdown is computed as the sum of all drawdown increments $s_{w(n)}=\Delta s_{w(1)}+\Delta s_{w(2)}+......+\Delta s_{w(n)}$. Eq. (10) then becomes:

$$\sum_{i=1}^{n} \Delta s_{w(i)} = s_{w(n)} = B(R_{e,} \Delta t)Q_n + CQ_n^2 \tag{11}$$

$\Delta s_{w(i)}$ = Drawdown increment between the ith step and the step preceding it [L]

If we divide both sides of Eq. (11) by Q_n:

$$\frac{s_{w(n)}}{Q_n} = B(R_{e,} \Delta t) + CQ_n \tag{12}$$

On arithmetic paper $s_{w(n)}/Q_n$ values are marked on the vertical axis and Q_n values on the horizontal axis. A straight line is fitted to these points. The slope of the line gives the value of *C*. The intersection of the fitted line with the vertical axis gives the value of *B*. Finally, the discharge-drawdown curve is obtained from Eq. (8) by using the values of *B* and *C*.

3.2.1.2. Rorabaugh method

This method is applied under the following assumptions [11], [3];

1. The aquifer is confined, leaky or unconfined.
2. The aquifer has an infinite areal extent.
3. The aquifer is homogeneous, isotropic and of uniform thickness.
4. Before pumping, the piezometric surface is horizontal over the area that will be influenced by the test.
5. The pumping rate at each step is held constant and greater than that of the previous step.
6. The well penetrates the entire thickness of the aquifer and receives water by horizontal flow.
7. The flow to the well is in an unsteady state.
8. The non-linear well losses can not be neglected and vary according to the expression CQ^P.

The total drawdown can be expressed as in the following when the superposition principle is applied to the Rorabaugh equation (Eq. (9)).

$$\sum_{i=1}^{n} \Delta s_{w(i)} = s_{w(n)} = BQ_n + CQ_n^P \tag{13}$$

If we divide both sides of Eq.(13) by Q_n:

$$\frac{s_{w(n)}}{Q_n} = B + CQ_n^{P-1} \tag{14}$$

When the logarithm of both sides are taken:

$$log\left(\frac{s_{w(n)}}{Q_n}-B\right)=logC+(P-1)\log Q_n \quad (15)$$

On a semi-logarithmic graph, time is placed to the horizontal logarithmic axis and the observed drawdown to the vertical axis. Drawdown increments are determined as in the previous method. The total drawdown is computed as the sum of all drawdown increments, $s_{w(n)}=\Delta s_{w(1)}+\Delta s_{w(2)}+_{......}+\Delta s_{w(n)}$. A value of B_i is assumed and the values of ($s_{w(n)}/Q_n$–B_i) is calculated for n varying from 1 to the last step. On a log-log paper ($s_{w(n)}/Q_n$–B_i) values are placed on the vertical axis and Q_n on the horizontal axis. The data points are connected with lines. The same procedure is repeated for different estimates of B_i. The value of B_i that gives the straightest line is the correct value of B. The slope of this line is equal to (P-1) from which P is obtained. The straight line is intersected with a vertical line passing through Q_n =1. The ordinate of this point gives the value of C. Finally, the discharge-drawdown curve is obtained from Eq. (9) using the values of B, C and P.

3.2.2. Recovery test

Recovery test is performed to obtain aquifer's hydraulic conductivity. Pumping is performed in one single well. After the pumping is ceased, water level tends to rise to the initial level. During the rise of the water level, the difference between the water level and initial level is called "residual drawdown". The test is applicable to confined aquifers with wells fully penetrating the aquifer and pumping at a constant rate. However, when certain conditions are satisfied it can be used in leaky, unconfined aquifers and in wells partially penetrating the aquifer [3]. Residual drawdown is expressed as follows [15]:

$$s'=\frac{Q}{4\pi T}\left[W(u)-W(u')\right] \quad (16)$$

s'= Residual drawdown [L]

Theis well function is defined as:

$$W(u)=-0.5772-lnu+u-\frac{u^2}{2*2!}+\cdots \quad (17)$$

u and u' in Eq.(16) are defined as:

$$u=\frac{r^2\,S}{4Tt}\ \text{ve}\ u'=\frac{r^2S'}{4T\,t'} \quad (18)$$

If u and u' are small enough (<0.01), the approximate value of residual drawdown becomes:

$$s'=\frac{Q}{4\pi T}\left[ln\frac{4Tt}{r^2S}-ln\frac{4Tt'}{r^2S'}\right] \quad (19)$$

r= Distance between the observation well and pumping well [L]

S'= Storativity during recovery [-]

S= Storativity during pumping [-]

t'= Time since the end of pumping [T]

Q= Pumping rate [L^3T^{-1}]

T= Transmissivity of the aquifer [L^2T^{-1}]

If $S = S'$ and T is constant,

$$s' = \frac{2.3Q}{4\pi T} \log\left(\frac{t}{t'}\right) \tag{20}$$

On a semi-log paper s' values are placed on the vertical axis and t/t' values to the horizontal logarithmic axis. A straight line is fitted to the plotted points. The slope of this line can be calculated as the residual drawdown difference $\Delta s'$ over one log cycle:

$$\Delta s' = \frac{2.3Q}{4\pi T} \tag{21}$$

from which the only unknown T is calculated. If T is divided by the saturated thickness, K can be obtained.

3.2.3. Multirate test

Multirate test is performed to determine aquifer's hydraulic conductivity [16]. For confined aquifers, Theis equation [15] is given as follows:

$$s = \frac{Q}{4\pi T} W(u) \tag{22}$$

The well function is defined in Eq. (17) and if $u<0.01$ then the drawdown in the well [17] can be expressed as:

$$s = \frac{2.3Q}{4\pi T} \log \frac{2.25Tt}{R_e^2 S} \tag{23}$$

s= Drawdown [L]

R_e= Effective radius of the well [L]

t= Time since the start of pumping [T]

Q= Pumping rate [L^3T^{-1}]

T= Transmissivity of the aquifer [L^2T^{-1}]

S= Storativity [-]

$W(u)$= Theis well function [-]

When the well loss is added to Eq. (23), the following is obtained:

$$s = \frac{2.3Q}{4\pi T}\log\frac{2.25Tt}{R_e^2\,S} + CQ^2 \tag{24}$$

where *C* is the well loss coefficient [18]. Eq. (24) can also be expressed as follows:

$$s = a(b + \log t)Q + CQ^2 \tag{25}$$

in which

$$a = \frac{2.3}{4\pi T} \tag{26}$$

$$b = \log\frac{2.25T}{R_e^2\,S} \tag{27}$$

In multirate test, several tests are performed with different pumping rates (Q_1, Q_2, …) and after each test a sufficiently long time is required for the water level to reach the pre-pumping condition. The drawdown is defined as a function of time for each test as in the following expression:

$$s_n = a(b + \log t)Q_n + CQ_n^2 \tag{28}$$

When both sides of Eq. (28) is divided by Q_n :

$$s_n/Q_n = a(b + \log t) + CQ_n \tag{29}$$

or

$$s_n/Q_n = A + CQ_n \tag{30}$$

After several arrangements:

$$A_n - A_{n-i} = a(b + \log t_n) - a(b + \log t_{n-i}) \tag{31}$$

$$= a\ \log\left(\frac{t_n}{t_{n-i}}\right) \tag{32}$$

$$= \frac{2.3}{4\pi T}\log\left(\frac{t_n}{t_{n-i}}\right) \tag{33}$$

from which *T* can be calculated:

$$T = \frac{2.3\ \log\left(\frac{t_n}{t_{n-i}}\right)}{4\pi(A_n - A_{n-i})} \tag{34}$$

On an arithmetic paper with s_n/Q_n on the vertical axis and Q_n on the horizontal axis for a selected *t* value. A straight line is fitted to the plotted points. This procedure is repeated for different *t* values and several lines are plotted. The slope of a line is *C*, and it intersects the vertical axis at A. The lines will have similar slopes, but different intercepts on the vertical axis. For two of the lines having the closest slopes, A and t are selected. Transmissivity is obtained from Eq. (34). Hydraulic conductivity can be calculated by dividing T by the saturated thickness. The test can be applied to both confined and unconfined aquifers. For unconfined aquifers, a correction [19] has to be made to the observed drawdown s as in Eq. (35):

$$s^* = s - s^2/2D \tag{35}$$

s^* = Corrected drawdown [L]

D= Saturated thickness of aquifer before pumping [L]

Field tests were solved using the analytical methods given above and the results were compared to those of MODFLOW and presented in the following section.

4. NUMERICAL MODELLING AND APPLICATION

All of the well tests performed in the field were modeled using the MODFLOW software [2]. MODFLOW is U.S. Geological Survey's modular 3-dimensional numerical groundwater model with a widespread user network. MODFLOW solves the governing partial differential equations of groundwater flow using method of finite differences.

4.1. Pumping Tests

For the well in which pumping tests were performed, a 200x200 grid with a total of 40000 active cells were generated for the numerical solution. An unconfined aquifer with 27 m depth is modeled and the flow is specified as transient. Time unit was specified as seconds, length unit was meters and flow rate unit was specified as m^3/s. Three different cases were tested during the model establishment.

Case 1: Cell size was fixed to 1 m x 1 m. As the aquifer consists of fine sand and silt material, specific yield was specified as S_Y =0.1 in all of the cells.

Case 2: Cell size was fixed to 0.117 m x 0.117 m which was equal to the well area. Specific yield was specified as S_Y =0.1.

Case 3: Cell size is fixed to 0.117 m x 0.117 m and specific yield was taken as S_Y =1 in the well and S_Y =0.1 in all of the other cells. A value of S_Y =1 is thought to give a similar drawdown in the well cell as in the real case.

In the Well package of MODFLOW, negative flow rates were specified for the pumping rates of each step. When no boundary condition is specified in MODFLOW, impervious boundary is automatically specified at the outer edge of the grid. In order to represent an infinitely large aquifer, the grid has to extent over a sufficiently large distance. In this study, the modeled area was: 200 m x 200 m for case 1, 23.4 m x 23.4 m for case 2 and 3. It was observed that both grids were large enough that the water levels remained unaffected by the impervious boundaries during the entire simulation. A homogeneous medium is assumed and one single hydraulic conductivity zone is specified. In a transient MODFLOW simulation, the duration within which a variable (e.g. pumping rate) is constant is called a stress period. Stress periods are defined by user. For example, in modeling the step-drawdown test, 4 stress periods were defined (Fig. 5). In order to capture the sudden change in drawdown at the beginning of each stress period, a multiplier such as 1.1 is used to specify gradually increasing time steps, (Δt), within a stress period. The hydraulic head measurements were specified as observation data and the value of hydraulic conductivity was tried to be estimated using *Parameter Estimation* model. During a MODFLOW run, the value of hydraulic conductivity is modified within predefined limits in order to minimize the error between the simulated and observed heads. The error criterion used is the Sum of Squared Weighted Residual (SSWR). The numerical results of the step-drawdown tests are presented in Table 3 and Fig. 5. Those of the pumping tests are presented in Table 4 and Fig. 6. As a result of analytical solutions, hydraulic conductivity was determined to be 3.60E-05 m/s from multirate test and 1.96E-05 m/s from recovery test. In this study, analytical methods and MODFLOW were used as two independent methods. As performing a MODFLOW simulation with a hydraulic conductivity value specified by the user will result in a larger error, the values obtained from analytical solutions were not used in numerical modeling.

Table 3. MODFLOW results for the step-drawdown test for 3 different cases

Date	Case	MODFLOW K (m/s)	MODFLOW SSWR
2.11.2012	1	2.52E-05	0.2510
	2	4.22E-05	0.1050
	3	4.20E-05	0.0944

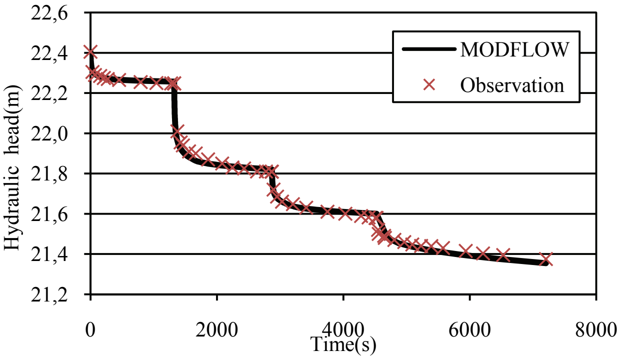

Figure 5. Comparison of MODFLOW results and field observations of the step-drawdown test (Discharges used are 0.17, 0.65, 0.85, 1.01 L/s)

Table 4. MODFLOW results for the pumping tests for 3 different cases

Date	Case	Q (m^3/s)	MODFLOW K (m/s)	MODFLOW SSWR
7.02.2013	1	6.79E-04	2.05E-05	0.88600
	2	6.79E-04	4.07E-05	0.45000
	3	6.79E-04	3.99E-05	0.19000
31.10.2012	1	5.38E-04	1.78E-05	0.55600
	2	5.38E-04	3.72E-05	0.02320
	3	5.38E-04	3.61E-05	0.02150
21.02.2013	1	3.52E-04	2.06E-05	0.11700
	2	3.52E-04	4.61E-05	0.03300
	3	3.52E-04	4.46E-05	0.01750
17.02.2013	1	3.16E-04	2.21E-05	0.10700
	2	3.16E-04	4.33E-05	0.13900
	3	3.16E-04	4.10E-05	0.10300
2.11.2012	1	1.71E-04	2.01E-05	0.02580
	2	1.71E-04	4.13E-05	0.00059
	3	1.71E-04	4.06E-05	0.00054

Simulations with Case 1 yielded the highest error values which were probably due to a large cell size used. For this reason, a study was performed in order to examine the effect of cell size on the overall results. Step-drawdown test was modeled using different cell sizes and it was observed that SSWR was decreasing with decreasing cell size (Fig. 7). Therefore, the analyses were performed based on Case 3. The reason for using 0.117 m x 0.117 m as a cell size is that it is equal to the area of the well and yields a reasonably low SSWR value.

Hantush-Bierschenk and Rorabaugh analytical methods were applied to step-drawdown test and discharge-drawdown curves were obtained as shown in Fig. 8. The curves almost coincide at low discharges and they fall apart as the discharge increases. Considering that the highest pumping rate used in this particular well was 1 L/s, both methods gave similar results up to 2 L/s. Higher discharges represent only an estimate of steady-state drawdown value.

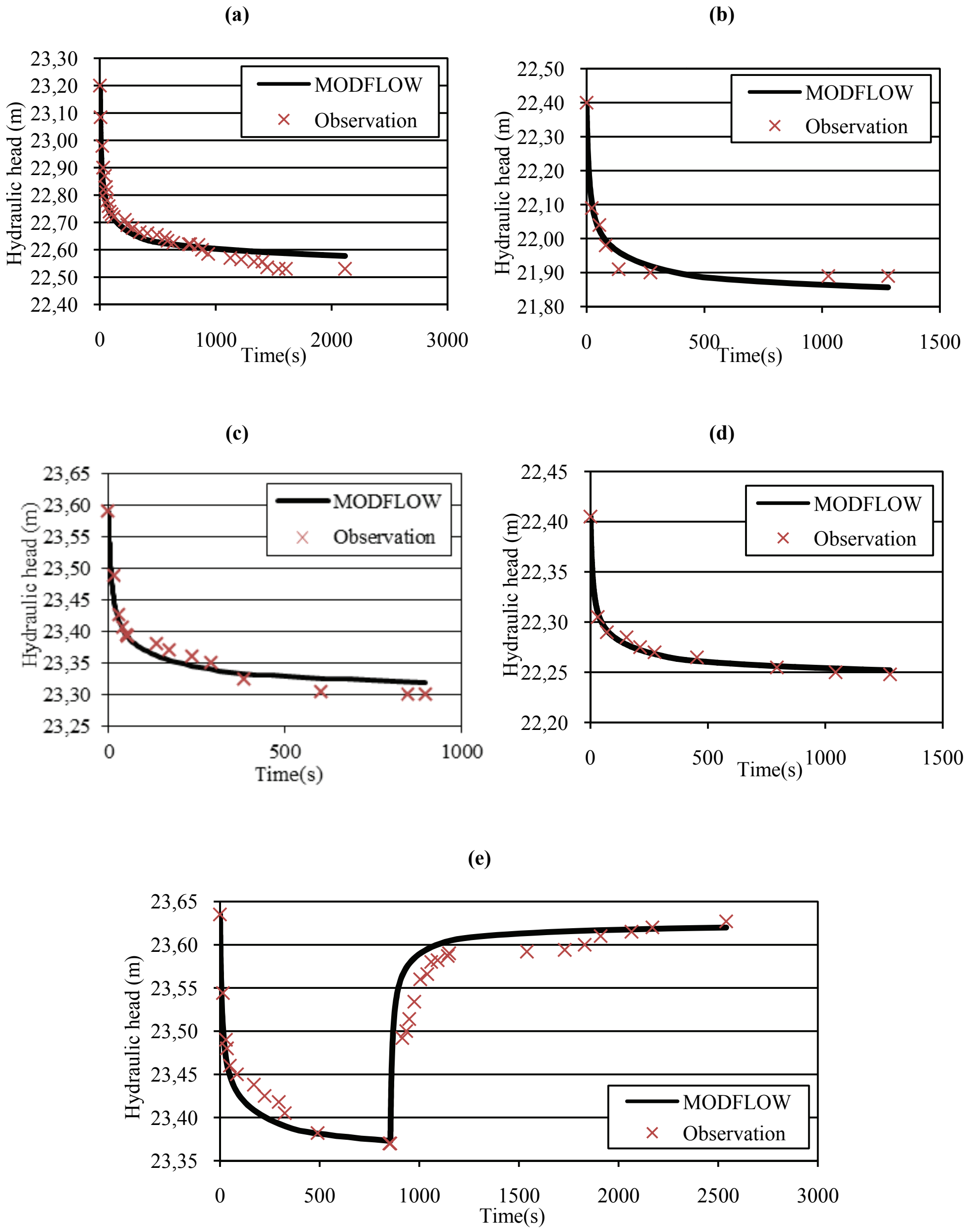

Figure 6. Comparison of MODFLOW results and field observations (a)7.2.2013 (b)31.10.2012 (c)21.2. 2013 (d)2.11.2012 (e)17.2.2013 (recovery test)

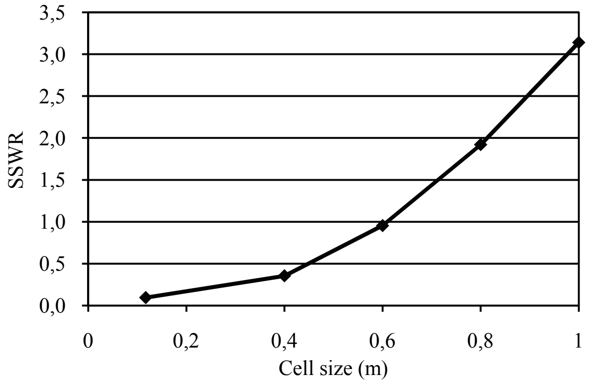

Figure 7. Variation of error value with cell size

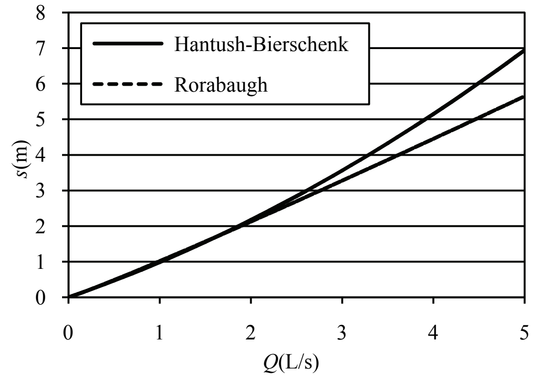

Figure 8. Discharge-drawdown curves obtained from step-drawdown test

4.2. Slug Tests

For modeling slug tests, a grid consisting of 200 rows and 200 columns with a total of 40000 active cells was generated in MODFLOW. In modeling, Case 3 was used for similar reasons as described above. An unconfined aquifer with a depth of 20 m and an unsteady flow were modeled. In order to simulate the sudden release of water into the well, the Well package was used and a positive pumping rate was specified. Therefore, slug test was modeled like injecting water into the well within a very short time interval. Thus, 2 stress periods were defined in slug tests. It was observed that modeled area (23.4 m x 23.4 m) was large enough that the water levels remained unaffected by the impervious boundaries during the entire simulation. A homogeneous medium was assumed and a single hydraulic conductivity zone was specified. Similar to pumping tests *Parameter Estimation* option was used to obtain the unknown hydraulic conductivity value. Hydraulic conductivity values solved by both analytical methods and MODFLOW are presented in Table 5. Arithmetic means of hydraulic conductivities are taken, and the results for different volumes and different methods are shown in Fig. 9.

Table 5. Analytical solutions and MODFLOW results for slug tests

Date	Well-development	Volume of water (L)	Bouwer-Rice *K*(m/s)	Dagan *K*(m/s)	MODFLOW *K*(m/s)
30.01.2013	not made	7	2.27E-06	2.38E-06	2.77E-06
30.01.2013	not made	7	2.22E-06	2.33E-06	3.44E-06
30.01.2013	not made	7	2.38E-06	2.49E-06	2.38E-06
15.01.2013	not made	10	2.28E-06	2.38E-06	1.09E-06
15.01.2013	not made	10	2.18E-06	2.28E-06	1.34E-06
30.01.2013	not made	10	2.56E-06	2.69E-06	1.85E-06
17.01.2013	not made	19	1.82E-06	1.90E-06	1.94E-06
17.01.2013	not made	19	2.78E-06	2.91E-06	2.08E-06
17.01.2013	not made	19	3.56E-06	3.74E-06	3.22E-06
17.02.2013	made	7	9.52E-06	9.97E-06	2.29E-05
17.02.2013	made	7	1.25E-05	1.31E-05	2.25E-05
17.02.2013	made	7	1.05E-05	1.10E-05	2.54E-05
17.02.2013	made	10	9.61E-06	1.01E-05	2.08E-05
17.02.2013	made	10	9.70E-06	1.02E-05	2.11E-05
17.02.2013	made	10	9.80E-06	1.03E-05	2.93E-05
17.02.2013	made	19	1.18E-05	1.23E-05	2.05E-05
17.02.2013	made	19	1.11E-05	1.16E-05	2.38E-05
17.02.2013	made	19	1.25E-05	1.31E-05	3.27E-05

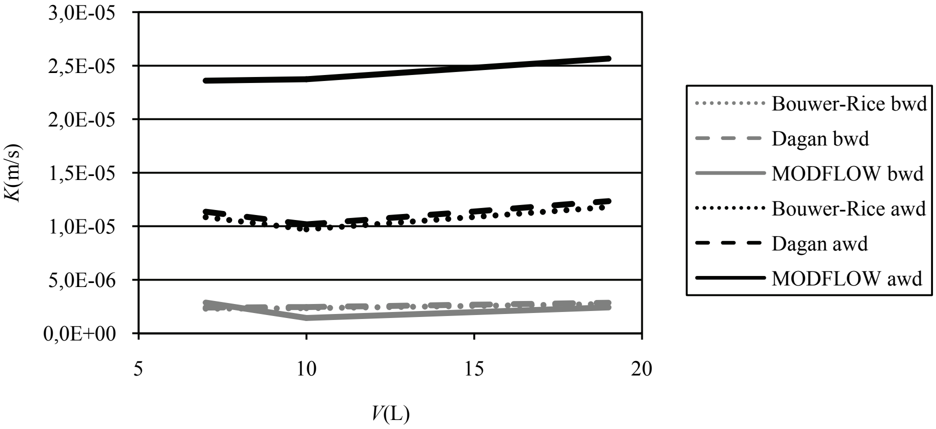

Figure 9. Comparison of analytical and MODFLOW solutions for slug tests performed before and after well development using 7, 10 and 19 L (bwd: before well development, awd: after well development)

5. RESULTS

In this study, well and aquifer tests were performed in two wells in an unconfined aquifer. Near-well hydraulic conductivity was determined through analytical and numerical methods applied to unsteady-state hydraulic head data. Mean values of hydraulic conductivity calculated with analytical and numerical methods are summarized in Table 6. It was observed that hydraulic conductivity calculated with analytical methods varies according to the method used. The reason is that analytical methods are derived for ideal cases and depend on certain conditions such as depth of well, length of well screen and the structure of well.

Table 6. Hydraulic conductivity values calculated with analytical and numerical methods (Arithmetic mean is taken for values calculated with the same method)

Well	Test name	Analytical Method K(m/s)		Numerical Method K(m/s)(case 3)
		Bouwer-Rice method	Dagan method	
2	Slug test-before well development	2.45E-06	2.56E-06	2.23E-06
	Slug test-after well development	1.08E-05	1.13E-05	2.43E-05
1	Recovery test	1.96E-05		4.10E-05
1	Multirate test	3.60E-05		4.04E-05
1	Step-drawdown test	-		4.20E-05

As seen from Tables 3 and 4, hydraulic conductivity values are close to each other in all pumping tests for case 2 and case 3. In addition to this, case 3 slightly gives better results. The reason is that, specific yield was taken equal to 1 in the cell in which the well was specified, and thus it represents the actual condition. Also, it was observed that pumping rate did not affect the hydraulic conductivity. For this reason, a realistic value of hydraulic conductivity can be reached through tests performed with low pumping rates. The analysis regarding the effect of grid resolution demonstrated that reducing cell size contributes to the accuracy of results (Fig. 7). When the numerical model is employed, the effect of grid resolution should be investigated especially in such local studies. The better the numerical model represents the actual field condition, the more realistic the results will be. In this study, as the difference of the hydraulic heads computed by the numerical model and field measurements is small, it is believed that the numerical model represents the actual situation successfully. Discharge-drawdown curves were obtained with Hantush-Bierschenk and Rorabaugh analytical methods applied to step-drawdown test data (Fig. 8). Such curves give ideas of what the drawdown can be in the event that a higher capacity pump is used.

According to the results of slug tests, both the Bouwer-Rice and Dagan analytical methods, and MODFLOW solution demonstrate that near-well hydraulic conductivity increases dramatically after well development. In addition to this, it was observed that the volume used in slug tests did not affect the results. The difference in results is probably due to the fact that ideal test conditions may not be fulfilled in the field. Well development process applied after drilling the well cleans the gravel envelope from clayey and sandy materials and hence increases the well efficiency. Therefore, it is crucial to apply well development in practice. Especially in industry, a demand can be supplied with less number of wells, the lifespan of wells and pumps can be extended, the quality of the pumped water increases and purification cost decreases. Due to all these reasons, economic benefit is gained.

Symbols

$B_1(r_c,t)$	Linear aquifer loss coefficient
B_2	Linear well loss coefficient
C	Non-linear well loss coefficient
D	Thickness of the saturated part of the aquifer
H_0	Change in water level right after pouring water (at t=0)
$H(t)$	Drawdown as a function of time
K	Hydraulic conductivity
L_e	Length of the screen
L_w	Distance between the water table and bottom of well
P	Dimensionless parameter
Q	Pumping rate
Q_i	Pumping rate during the ith step
Q_n	Pumping rate during the nth step
R_e	Effective radius of well
r	Distance between the observation well and pumping well
r_c	Radius of the well pipe
r_w	Radius of gravel envelope
S	Storativity during pumping
S'	Storativity during recovery
S_Y	Specific yield
s	Drawdown
s^*	Corrected drawdown

s'	Residual drawdown
$s_{w(n)}$	Total drawdown in the well during the n^{th} step at time t
T	Transmissivity of the aquifer
t	Time
t_i	Time at which the i^{th} step begins
t_{new}	Corrected time
t'	Time since the end of pumping
V	Volume of water poured
$W(u)$	Theis well function
X	Dimensionless parameter
Y	Dimensionless parameter
Z	Dimensionless parameter
$\Delta s_{w(i)}$	Drawdown increment between the i^{th} step and the step preceding it
ψ	Dimensionless parameter used in Dagan method

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